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The numerical value zero (for $\varphi=0$) is however unaltered by the group of the equation. (If φ were equal to $r(x)$ instead of to zero, the group of the equation would be (2) with $ad-bc=1$; for $ad-bc\neq1$, the numerical value would change from $r(x)$ to $(ad-bc)^3r(x)$.)

In the above example we assumed that the function $\varphi=y_2^2-y_1y_3$ was the invariant of the group. If the differential equation was of a higher order than the third the relation would be more complicated. Indeed, we might have a number of such relations involving not only the integrals but also their derivatives, say $\varphi_i(y_1\ldots.y_n,\ y_1'\ldots.y_n,\ y_1''\ldots.)$, $(i=1\ldots.p)$. The generalization can be carried still further. Notice that we assumed above that there exists algebraic relations between the integrals and their derivatives. Now this assumption is unnecessary, it may well happen that there are no such relations whatever, yet we continue still to speak of the group of the equation. An exposition of this question would be beyond the range set for this paper.

The group in question is technically called the *group of rationality* of the equation.

The literature of this subject is now quite extensive, but anyone familiar with the elements of the Galois theory will find it very easy to read the brief summary in Picard's *Traité de Analyse* III, last chapter, and to supplement it with Klein's *Höhere Geometrie* II, pp. 298-9.

The University of Chicago, October, 1902.

TWO SIMPLE CONSTRUCTIONS FOR FINDING THE FOCI OF AN HYPERBOLA, GIVEN THE ASYMPTOTES AND A POINT ON, OR A TANGENT TO, THE CURVE.

By ARCHIBALD HENDERSON, Ph. D.. Associate Professor of Mathematics, University of North Carolina.

The construction for a special position of the point is given first, as it is a linear construction.

Given the asymptotes of a hyperbola and the vertex A of the curve, to construct the foci.

The major axis is fixed, bisecting the angle between the asymptotes. Lay off, along an asymptote, from the center C of the hyperbola a distance $CD=CA=a$; draw a perpendicular to the asymptote at D. This meets the major axis (produced) in a focus $F_1\,(F_2)$.

For, the perpendicular F_1D from the focus $(ae,\,0)$ upon the asymptote, whose equation is $y=\frac{b}{a}x$, is equal to b (by elementary principles). But $CF_1=\sqrt{(a^2+b^2)}$ and therefore $CD=a$.

The following style of argument (communicated to me in a letter a few days ago), while ordinarily to be avoided, is rather interesting in this connection. Granting a previous knowledge of the property in question (such as given by the above proof), this second proof furnishes an example of the use of the indeterminate form $\infty-\infty=k$ (k=constant), in a geometrical problem.

Let P, a point on the curve, move out to infinity on the upper half of the right-hand branch of the hyperbola. From F_1, the right-hand focus, draw a perpendicular to CP_∞, meeting it at D and meeting F_2P_∞ at E.

Now, by the defining property of the hyperbola,

$$F_2P_\infty-F_1P_\infty=2a.$$

Therefore $F_2E=2a$, and consequently $CD=a$.

The general problem is the following: *Given the asymptotes of an hyperbola and a point P on, or a tangent DE to, the curve; to construct the foci.*

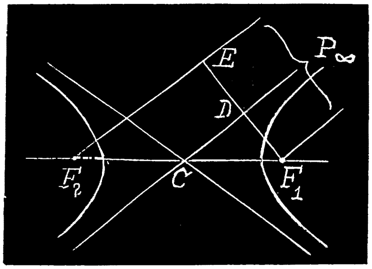

Fig. 1.

If we are given P, DE is determined; for, PG, drawn parallel to one (either) asymptote, meets the other at G, say. GE laid off equal to OG fixes the tangent DE in postion, since the segment of any tangent to an hyperbola intercepted between the asymptotes is bisected at the point of contact.

Hence only the case in which the tangent is given will be considered, since the other may be immediately reduced to it.

Construction. Draw PC perpendicular to the tangent DE at its middle point P, and let it intersect the minor axis at C. Then the circle with center C and radius $CD(=CE)$ cuts the major axis in the two foci F_1 and F_2.

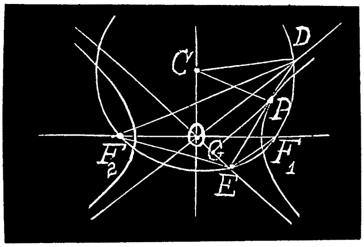

Fig. 2.

For, if P, D, C are designated respectively by (x_1, y_1), (x_2, y_2), $(0, y_3)$, we find by obvious methods that

$$y_3=\frac{a^2e^2y_1}{b^2},\quad x_2=\frac{bx_1+ay_1}{b},\quad y_2=\frac{bx_1+ay_1}{a},$$

and therefore, by the formula for the distance between two points, we obtain $CD=CF_1$

This may be very easily proved by projective geometry in the following manner. It is a well known property that the angle $EF_1D(EF_2D)$ is invariant for all positions of the variable tangent DE. Hence, when ED coincides with OD, $\angle EF_1D=\angle DOF_2$, $\angle EF_2D=\angle DOF_1$ (since when D moves out to infinity, F_1D_∞ and F_2D_∞ are parallel to OD. Accordingly,

$$\angle EF_1D+\angle EF_2D=180^\circ$$

and the points F_1, E, F_2, D are concyclic. The perpendicular bisectors of F_1F_2 and DE therefore intersect at C, the required center.

It may be observed that the second construction is a direct generalization of the one (when the asymptotes and vertex are given) found in all elementary text-books on Analytic Geometry. Would it not be well for the latter to contain the above simple constructions?

The University of Chicago, October, 1902.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

161. Proposed by F. M. SHIELDS, Coopwood, Miss.

If 1 man, 1 boy, and 1 girl catch 1 trout, 1 perch, and one minnow in 5 minutes, and 1 man, 2 boys, and 3 girls catch 1 trout, 2 perch, and 3 minnows in 6 minutes, how many minutes will be required for 2 men, 3 boys, and 4 girls to catch 5 trout, 11 perch, and 17 minnows?

Remarks by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

This problem is indeterminate. Let A_1, A_2, and A_3 be the activities of a man, boy, and girl, respectively; and E_1, E_2, and E_3 the work required to catch a trout, a perch, and a minnow, respectively. Then

$$5(A_1+A_2+A_3)=E_1+E_2+E_3\ldots.(1), \text{ and}$$
$$6(A_1+2A_2+3A_3)=E_1+2E_2+3E_3\ldots.(2).$$

From these two conditions it is required to find t so that the following condition is satisfied, viz:

$$t(2A_1+3A_2+4A_3)=5E_1+11E_2+17E_3.$$

This is clearly impossible in general. By assuming certain other conditions as, for example, the ratios of the activities, and the ratios of the work, a solution may be effected.

162. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

A trolly road is built between two towns, and it is found that the gross annual receipts amount to 20% of the original cost; the annual cost of repairs 2% of the original cost; and the working expense is $3000 in addition to 20% of the net receipts. After a year a second road is built at the same cost as the first and it is found that the gross re-